

A New Independent Limit on the Cosmological Constant/Dark Energy from the Relativistic Bending of Light by Galaxies and Clusters of Galaxies

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ABSTRACT

We derive new limits on the value of the cosmological constant, Λ , based on the Einstein bending of light by systems where the lens is a distant galaxy or a cluster of galaxies. We use an amended lens equation in which the contribution of Λ to the Einstein deflection angle is taken into account and use observations of Einstein radii around several lens systems. We use in our calculations a Schwarzschild-de Sitter vacuole exactly matched into a Friedmann-Robertson-Walker background and show that a Λ -contribution term appears in the deflection angle within the lens equation. We find that the contribution of the Λ -term to the bending angle is larger than the second-order term for many lens systems. Using these observations of bending angles, we derive new limits on the value of Λ . These limits constitute the best observational upper bound on Λ after cosmological constraints and are only two orders of magnitude away from the value determined by those cosmological constraints.

Key words: cosmology: theory – gravitational lensing – gravitation.

1 INTRODUCTION

Cosmic acceleration and the dark energy associated with it constitute one of the most important and challenging current problems in cosmology and all physics, see for example the reviews (Weinberg 1989; Turner 2000; Sahni & Starobinsky 2000; Carroll 2001; Padmanabhan 2003; Peebles & Ratra 2003; Upadhye et al. 2005; Albrecht et al. 2006; Ishak 2007) and references therein. The cosmological constant, Λ , is among the favored candidates responsible for this acceleration. Current constraints on Λ are coming from cosmology, see e.g. (Riess et al. 1998; Perlmutter et al. 1999; Knop et al. 2003; Riess et al. 2004; Bennett et al. 2003; Spergel et al. 2003; Page et al. 2003; Seljak et al. 2005; Tegmark et al. 2004; Spergel et al. 2007), and it is important to obtain constraints or limits from other astrophysical observations.

Very recently, the authors of reference (Rindler & Ishak 2007) demonstrated that, contrarily to previous claims (e.g. (Islam 1983; Freire et al. 2001; Kagramanova et al. 2006; Finelli et al. 2007; Sereno & Jetzer 2006; Kerr et al. 2003)), when the ge-

ometry of the Schwarzschild-de Sitter spacetime is taken into account, the cosmological constant does contribute to the light-bending around a concentrated source and hence to the corresponding Einstein deflection angle. This result was confirmed in (Lake 2007; Sereno 2007; Schucker 2007).

In this paper, we incorporate that result into the broadly used lens equation and then apply it to current observations of Einstein radii around distant galaxies and clusters of galaxies. Using observational data of a selected list of Einstein radii around clusters and galaxies, we show that the contribution of the cosmological constant to the bending angle can be larger than the second-order term of the Einstein bending angle. These new results allow us to put new independent upper bounds on the value of the cosmological constant based on the observations of the bending angle by galaxies and clusters of galaxies. The results provide an improvement of eight orders of magnitude on previous upper bounds on Λ from planetary or stellar systems, see for example (Sereno & Jetzer 2006; Kagramanova et al. 2006). Interestingly, these limits provide the best observational upper bound on Λ after cosmological constraints and are only two orders of magnitude away from the value determined by those cosmological constraints.

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$$\frac{d^2 \delta u_1}{d\phi^2} + \delta u_1 = 3 \sin^2 \phi \quad (11)$$

$$\frac{d^2 \delta u_2}{d\phi^2} + \delta u_2 = 6 \delta u_1 \sin \phi. \quad (12)$$

Solving (11) and (12) for δu_1 and δu_2 and substituting them into (10) gives the solution

$$\frac{1}{r} = \frac{\sin \phi}{R} + \frac{3m}{2R^2} \left(1 + \frac{\cos 2\phi}{3}\right) + \frac{3m^2}{16R^3} \left(10\pi \cos \phi - 20\phi \cos \phi - \sin 3\phi\right). \quad (13)$$

Now, we differentiate (13) and multiply by r^2 to obtain

$$\frac{dr}{d\phi} = -\frac{r^2}{R} \cos \phi + \frac{mr^2}{R^2} \sin 2\phi + \frac{15m^2 r^2}{4R^3} \left(\cos \phi + \frac{3}{20} \cos 3\phi + \left(\frac{\pi}{2} - \phi\right) \sin \phi\right). \quad (14)$$

After some manipulation, it follows from (7) and (14) that the total bending angle (at $\phi = 0$) to the third-order is given by

$$\alpha \approx 4 \frac{m}{R} + \frac{15\pi}{4} \frac{m^2}{R^2} + \frac{305}{12} \frac{m^3}{R^3} - \frac{\Lambda R^3}{6m}. \quad (15)$$

The coefficients for the first and second-order terms in this expansion are the same as the ones in the expansion in terms of the impact parameter b , see e.g. (Keeton & Petters), that is used for the asymptotically flat Schwarzschild spacetime. In the next section, we put our results into an observational context using systems where the lens is a galaxy or a cluster of galaxies.

3 OBSERVATIONS OF EINSTEIN-RADII AND THE CONTRIBUTION OF THE COSMOLOGICAL CONSTANT TO THE DEFLECTION

As one might expect, while the cosmological constant has a very negligible effect on small scales this is not the case at the level of galaxies and clusters of galaxies. In this section, we evaluate the contribution of the cosmological constant to the bending of light using observations of large Einstein radii where the lens is a galaxy or a cluster of galaxies.

Equations (9) and (15) above were derived based on a source and an observer located in a Schwarzschild-de Sitter background. We will derive here the corresponding equation in a Friedmann-Lemaître-Robertson-Walker background (FLRW). For that, we consider a Schwarzschild-de Sitter vacuole exactly embedded into an FLRW spacetime using the Israel-Darmois formalism (Darmois 1927; Israel 1966). The relations between radial coordinates r_b at the boundary of the vacuole are simple and well-known in the literature, see for example (Einstein & Strauss 1945; Schucking 1954), and are given by the following two equations:

$$r_{b \text{ in SdS}} = a(t) \ r_{b \text{ in FLRW}} \quad (16)$$

and

$$m_{\text{SdS}} = \frac{4\pi}{3} r_{b \text{ in SdS}}^3 \times \rho_{\text{matter in FLRW}}. \quad (17)$$

Thus, for a given cluster mass, equation (17) provides a boundary radius where the spacetime transitions from a SdS spacetime to an FLRW background. We shall assume that all the light-bending occurs in the SdS vacuole according to our previous formulae, and that once the light transitions out of the vacuole and into FLRW spacetime, all Λ -bending stops. Unlike the mass-effect, which falls off quickly, the Λ -effect on the bending of light increases with distance from the source (the " Λ -repulsion" is proportional to distance); hence the question of where to cut off the integration becomes important. The choice of the boundary of the vacuole (r_b) in the Einstein-Strauss model seems physically the most appropriate, whereas the choice $\phi = 0$ in ref. (Rindler & Ishak 2007) was purely conventional.

Now, for the small angle ϕ_b at the boundary, equation (5) gives

$$u_b = \frac{1}{r_b} = \frac{\phi_b}{R} + \frac{2m}{R^2} \quad (18)$$

and equation (8) gives

$$|A| = \frac{r_b^2}{R} \left(1 - \frac{2\phi_b m}{R}\right). \quad (19)$$

Next, inserting (18) and (19) into equation (7) yields after a few steps

$$\theta \approx \tan \theta \approx \phi_b + \frac{2m}{R} - \frac{\Lambda \phi_b r_b^2}{6} + \text{higher-order terms}. \quad (20)$$

The bending angle, α , is given, to the smallest order in m/R and Λ , by

$$\frac{\alpha}{2} \approx \theta - \phi_b \approx \frac{2m}{R} - \frac{\Lambda \phi_b r_b^2}{6}. \quad (21)$$

Now, equation (18) yields, to the smallest order, $\phi_b = R/r_b$, so we can finally write from (21)

$$\alpha \approx \frac{4m}{R} - \frac{\Lambda R r_b}{3} \quad (22)$$

where R is related to the closest approach by equation (6) and r_b is the boundary radius between SdS and FLRW, and is given by equation (17).

Perhaps a caveat that one need to address is that the Einstein-Strauss model that was used here is known to have some instability to radial perturbations at the boundary as, for example, discussed in (Krasinski 1997) and references therein. However, our work hinges on finding a cut-off location where the Λ -bending of the lens can be regarded as accomplished. In the predecessor paper to this one (Rindler & Ishak 2007) we chose $\phi = 0$ as the only readily available standard cut-off point. The present paper is an improvement over the previous one in this respect, in that we now have a cut-off point tailored to each individual lens, namely the edge of the vacuole. The vacuole model as such is not used except for this one purpose, namely to give us a realistic order-of-magnitude estimate of the range of influence of the lens. Moreover, as it is widely used in gravitational lensing studies, one could also resort to approximation methods where the inhomogeneity is modeled by a gravitational potential that is embedded in an FLRW background (Bartelmann & Schneider 2001; Mellier 1999; Carroll 2004). Such an alternative treatment of the questions addressed here has been recently carried out in (Ishak 2008) and has confirmed the findings of the present work.

Also, our result is expressed in terms of the vacuole boundary r_b that is evaluated at some instant in time. The vacuole and its boundary expand as the universe expands thus when we calculate r_b from equation (17) we must use the density of the universe as it was when light passed by the lens. We are aware of the instability of r_b but since we need it at one instant, the instability should not affect our result.

Next, using equations (13) and (14), we can expand the result to

$$\alpha \approx 4 \frac{m}{R} + \frac{15\pi}{4} \frac{m^2}{R^2} + \frac{305}{12} \frac{m^3}{R^3} - \frac{\Lambda R r_b}{3} \quad (23)$$

Finally, following the usual procedure, see e.g. (Mellier 1999; Bartelmann & Schneider 2001), we put our results into the lens equation which is given from the geometry (see Figure 1) and small-angle relations as follows

$$\theta D_{OS} = \beta D_{OS} + \alpha D_{LS} \quad (24)$$

or in the familiar form

$$\theta = \beta + \alpha \frac{D_{LS}}{D_{OS}} \quad (25)$$

where all the quantities are as defined in Figure 1, and the angular-diameter distance is given by

$$D(z) = \frac{c}{H_0(1+z)} \int_0^z \frac{dz'}{\sqrt{\Omega_m(1+z')^3 + \Omega_\Lambda}} \quad (26)$$

where, for the spatially flat concordance cosmology, $\Omega_m = 0.27$, $\Omega_\Lambda = 0.73$, and $H_0 = 71 \text{ km/s/Mpc}$.

Thanks to the advancement of observational techniques, one can find in the literature a number of distant galaxies and clusters of galaxies that are lenses with large Einstein radii, making them very interesting for applying our results. The selected systems are shown in Table 1 along with our evaluation of the deflection first-order term, the second-order term, and the Λ -term, and some of their ratios. Despite the smallness of the cosmological constant, Λ , we find that the Einstein first-order term in the bending

angle due to these systems is only by some 10^3 bigger than the Λ -term. Interestingly, we find that for the lens systems in Table 1, the contribution of the cosmological constant term is larger than the second-order term of the Einstein bending angle.

4 A NEW LIMIT ON THE COSMOLOGICAL CONSTANT FROM LIGHT-BENDING

From cosmology (e.g. using supernova magnitude-redshift relation and the Cosmic Microwave Background Radiation), the value of the cosmological constant, Λ , is found to be about $1.29 \times 10^{-56} \text{ cm}^{-2}$ (using $H_0 = 71 \text{ km/s/Mpc}$ and $\Omega_\Lambda = 0.73$, see e.g. (Rindler 1969; Riess et al. 1998; Perlmutter et al. 1999; Knop et al. 2003; Riess et al. 2004; Bennett et al. 2003; Spergel et al. 2003; Page et al. 2003; Seljak et al. 2005; Tegmark et al. 2004; Spergel et al. 2007)). It is very desirable to obtain other limits on Λ that come from other astrophysical constraints. As we show, when we consider the uncertainty in the measurements of the bending angle (which is around $\Delta\alpha \sim 5\text{-}10\%$ for several of the systems considered in Table 1), we find

that the bending angle due to distant galaxies and clusters can provide interesting limits on the value of the cosmological constant. Indeed, if the contribution of Λ cannot exceed the uncertainty in the bending angle for these system, then it follows that

$$\Lambda \leq \frac{3 \Delta\alpha}{R r_b}. \quad (27)$$

For example, with $\Delta\alpha = 10\%$, we find from the system Abell 2744 (Smail et al. 1991; Allen 1998) that

$$\Lambda \leq 4.23 \times 10^{-54} \text{ cm}^{-2}. \quad (28)$$

The other limits are in Table 1. Interestingly, these limits are the best observational upper bound on the value of Λ after cosmological constraints and are only two orders of magnitude away from the value determined from cosmological constraints. In fact, Λ also enters into the expression of the angular diameter distance but our estimation is that it can affect our limit by a factor of two or less. Previously, the best upper bound after cosmology was provided from planetary or stellar systems and is $\Lambda \leq 10^{-46} \text{ cm}^{-2}$, see for example (Serenio & Jetzer 2006; Kagramanova et al. 2006) and references therein.

5 CONCLUSION

In conclusion, we showed that a Λ -contribution term appears in the deflection angle within the lens equation. This contribution is larger than the second-order term in the Einstein bending angle for many cluster lens systems. These results allow us to put new upper bounds on the cosmological constant, Λ , based on observations of the bending angle by galaxies and clusters of galaxies. These results provide an improvement of 8 orders of magnitude on previous upper bounds on Λ that were based on planetary or stellar systems, e.g. (Serenio & Jetzer 2006; Kagramanova et al. 2006). The limits provide the best upper bound on Λ after cosmological constraints and are only two orders of magnitude away from the value determined for Λ from those cosmological constraints.

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Table 1. Contributions of the cosmological constant to the Einstein bending angle by distant clusters of galaxies. Column-8 shows that the Λ -term contribution is larger than the second-order term in the Einstein bending angle for these lens systems. The last column shows limits on the cosmological constant based on observations of the bending angle. These limits provide the best upper bound on Λ after cosmological constraints and are only two orders of magnitude away from the value determined for Λ by those cosmological constraints, i.e. $1.29 \times 10^{-56} \text{cm}^{-2}$. Previously, the best upper bound after cosmology was determined from planetary or stellar systems and is $\Lambda \leq 10^{-46} \text{cm}^{-2}$, see for example (Serenio & Jetzer 2006, Kagramanova et al. 2006) and references therein.

Cluster or galaxy name and references	Einstein Radius (Kpc)	Mass in $M_{\text{sun}} h^{-1}$	1st Order term (rads)	2nd Order term (rads)	Λ -term (rads)	Ratio-1 1st/ Λ -term	Ratio-2 Λ -term/2nd	Upper Limit on Λ (cm^{-2})
Abell 2744 (Smail et al. 1991; Allen 1998)	96.4	1.97×10^{13}	5.53E-05	2.25E-09	1.68E-08	3.28E+03	7.48	4.23E-54
Abell 1689 (Allen 1998; Limousin 2007)	138.2	9.36×10^{13}	1.88E-04	2.61E-08	4.52E-08	3.52E+03	1.73	5.37E-54
SDSS J1004+4112 (Sharon 2006)	110.0	4.26×10^{13}	1.05E-04	8.06E-09	2.22E-08	4.70E+03	2.76	6.07E-54
3C 295 (Wold et al. 2002)	127.7	7.1×10^{13}	1.50E-04	1.66E-08	3.06E-08	4.90E+03	1.84	6.33E-54
Abell 2219L (Smail et al. 1995a; Allen 1998)	86.3	3.22×10^{13}	1.01E-04	7.47E-09	1.85E-08	5.44E+03	2.48	7.01E-54
AC 114 (Smail et al. 1995b; Allen 1998)	54.6	9.23×10^{12}	4.57E-05	1.54E-09	7.38E-09	6.19E+03	4.80	7.99E-54

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